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ON THE DISPERSING PROPERTIES OF A MEDIUM IN THE PRESENCE
OF VERY HIGH DENSITIES AND TEMPERATURES

REPORT II

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- USSR -

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ON THE DISPERSING PROPERTIES OF A MEDIUM IN THE PRESENCE
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REPORT II

[Following is the translation of an article by G. S. Saakyan, entitled "O Dispersionnykh Svoystvakh Sredy pri Ochen' Bol'sikh Plotnotyakh i Temperaturakh, Soobshcheniye II" (English version above) in Doklady Akademii Nauk Armyanskoy SSR (Reports of the Academy of Sciences Armenian SSR), Vol. 30, No. 4, 1960, Yerevan, pages 211-218.]

(Presented by corresponding member of the AS Armenian SSR, N. M. Kocharyan, 15 June 1959)

Dispersion of waves on nucleons. Under definite physical conditions, namely at sufficiently high densities of the substance, sufficiently low temperatures (so that the electron gas can be considered completely degenerate), and sufficiently high frequencies, the dispersion of electromagnetic waves takes place not on electrons, but on nucleons. Under such physical conditions the phenomenon of hard Cerenkov radiation inevitably arises.

To explain the essence of the matter, we shall investigate the dispersive properties of a medium consisting of neutrons. Let us calculate the index of refraction of electromagnetic waves for each medium and consider at which frequencies and densities of the neutrons

the dispersive properties of the medium will be determined not by the electrons, but by the neutrons (in general we should speak of nucleons; however the number of protons under the physical conditions which interest us is small in comparison with the number of neutrons, and hence we shall be able to speak later of the latter; however, all the conclusions we obtain will pertain equally to protons as well).

Notwithstanding the fact that neutrons are electrically neutral, nonetheless they are capable of dispersing γ -quanta. In fact it is experimentally known that γ -quanta can be absorbed by neutrons, giving rise to neutral and negative π^0 -mesons. This is the well known phenomenon of photogeneration of mesons. On the other hand, every system capable of absorbing electromagnetic radiation must of necessity disperse them also.

The ability of neutrons to absorb and disperse electromagnetic waves is determined by their electromagnetic structure, i.e., by the existence of a virtual meson cloud around the nucleon. When photons interact with neutrons, both virtual and real mesons can be generated. The real generation of mesons produces absorption of the wave, and hence the refractive index of a medium consisting of neutrons will be a complex quantity,

$$\sqrt{\epsilon} = n + i n_1, \quad (1)$$

where ϵ is the dielectric constant of the medium, n_1 determines the absorption of the wave, and n is the rate of its propagation in the

medium. The relationship [1]

$$n(\omega) = 1 + \frac{2}{\pi} \int_0^\infty \frac{x n_1(x)}{x^2 - \omega^2} dx, \quad (2)$$

where the vinculum indicates that the integral should be taken in the sense of the principal value, exists between the real and imaginary portions of the index of refraction.

Let $\sigma'(\omega)$ be the total cross section of photogeneration of mesons. Then the coefficient of absorption of γ -quanta in a neutron substance is equal to $N_n \sigma'$, where N_n is the number of neutrons in a unit of volume. On the other hand, this coefficient of absorption is equal to $2\omega/c n_1$; consequently

$$n_1(\omega) = \frac{c N_n \sigma(\omega)}{2\omega}. \quad (3)$$

From (3) and (2) we find,

$$n(\omega) = 1 + \frac{c N_n}{\pi} \int_0^\infty \frac{\sigma(x) dx}{x^2 - \omega^2}. \quad (4)$$

At present there is no accurate theory of photogeneration. In reference [2] a phenomenological theory was constructed, based on the hypothesis [3], [4] of the existence of an isobaric state of a nucleon with quantum numbers $I = 3/2$ and $T = 3/2$ (I is the total moment of inertia and T is the total isotopic spin) and with the resonance level $E_0 = 154$ mev. At present this hypothesis has been confirmed by experiments. In addition, the experiments indicate the presence

of two other isobaric states ($3/2$, $1/2$) and ($3/2$, $5/2$), with higher levels of the energy of excitation. The dispersion formula of Cell-Mann and Watson [2] satisfactorily describes the experimental data up to an energy of 400 mev. However, it has an unwieldy form and is unsuitable in the sense of its application for calculation of $n(\omega)$.

We shall proceed from the experimental curve of photogeneration of π^0 -mesons on protons, depicted in Fig. 1. This curve has two maxima: the first, very sharp and well defined at $h\omega \approx 320$ mev, corresponds to the first isobar of the nucleon $3/2$, $3/2$, while the second, weakly defined and broad at $E \approx 700$ mev, corresponds to the isobar $3/2$, $1/2$.

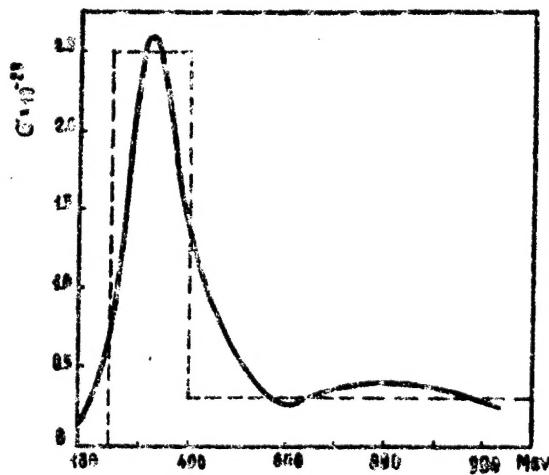


Fig. 1

As we move to the center of a star the density of neutrons increases and according to condition (1) the region of applicability of

taking the index of refraction in the direction of high energies of the quanta correspondingly broadens. In the limiting case we are dealing with the nuclear density. The average distance between particles in this case is equal to $h/\mu c$ (μ is the mass of the π -meson) and the corresponding limiting energy of the photons should be of the order of 150 mev. Hence we should speak of the index of refraction of a neutron substance only at energies of the quanta of $\pi\omega \leq 150$ mev. In this energy region $\sigma'(\omega) = 0$, while the principal maximum of this function is located at an energy two times in excess of 150 mev. It is clear from this that inasmuch as the frequencies $\omega \leq 150$ mev are of interest to us, then to determine $n(\omega)$ a detailed picture of the dependence of $\sigma'(\omega)$ on ω will not be necessary. Hence, as a good approximation we can replace the true curve of Fig. 1 by the broken line shown by a dotted line. Here the dotted line is selected in such a way that the area formed by it with the x axis approximately coincides with the area formed by the experimental curve. Let us further assume that the cross sections of photogeneration of π^0 - and π^+ -mesons on protons are approximately equal (which actually is approximately the case ^{4,5,6,7}), and that sections of photogeneration on neutrons and protons are also equal. Thus for $\sigma'(\omega)$ it is assumed that

$$\sigma(\omega) = \begin{cases} 0 & \text{при } \omega < \omega_1 \\ \sigma_1 & \text{при } \omega_1 < \omega < \omega_2 \\ \sigma_2 & \text{при } \omega > \omega_2, \end{cases} \quad (5)$$

where $\sigma_1 = 5 \cdot 10^{-28} \text{ cm}^2$; $\sigma_2 = 0.62 \cdot 10^{-28} \text{ cm}^2$; $\omega_1 = 3.7 \cdot 10^{23}$ is the frequency corresponding to an energy of the photons of 240 mev; and finally $\omega_2 = 6.1 \cdot 10^{23}$ is the same thing for an energy of 400 mev.

Substituting (5) into (4) and performing the integration, we obtain

$$n^2 - 1 = \frac{cN_n\sigma_1}{\pi\omega} \ln \frac{\omega_1 + \omega}{\omega_1 - \omega} + \frac{cN_n(\sigma_2 - \sigma_1)}{\pi\omega} \ln \frac{\omega_2 + \omega}{\omega_2 - \omega}. \quad (6)$$

here, assuming $\omega \ll \omega_1$ (let us recall that selection of the index of refraction in the limiting case of nuclear densities $\approx 10^{38}$ was limited by the energies of the quanta $E\omega \leq 150$ mev, while $\hbar\omega_1 = 240$ mev), we obtain

$$n^2 - 1 = \frac{2cN_n}{\pi} \left(\frac{\sigma_1}{\omega_1} - \frac{\sigma_1 - \sigma_2}{\omega_2} \right) = 2.1 \cdot 10^{-41} N_n. \quad (7)$$

Thus the dielectric constant of a neutron substance is practically independent of the frequency and differs very little from one.

In order to explain in the final calculation by which the dispersive properties of the medium are determined: whether by electrons or by neutrons, (7) should be compared with (22). * The dispersion

* See Report I, DANArSSR (Reports of the AS ArSSR), Vol. XXX, No. 1 (1960).

will be determined by neutrons in the case when

$$\frac{n^2 - 1}{1 - n_e} = \frac{2.1 \cdot 10^{-4} N_n}{\omega_0^2} \omega^2 > 1, \quad (8)$$

where the index "e" refers to the electron. From (23), substituting the value of ω_0 , we find

$$\omega^2 N_n > 0.94 \cdot 10^{60} N_e^{1/3}. \quad (9)$$

Proceeding from this formula, we can explain what possibilities exist in the case when the electronic gas is not fully degenerate. First let us consider the case when a nucleonic gas is not degenerate. In order for the dispersion of the waves to be determined by electrons according to (30)* and (9), we should have

$$N_e^{1/3} \exp\left(\frac{a}{T} N_e^{1/3}\right) < 3.6 \cdot 10^{17} \left(\frac{\omega_0}{\omega}\right)^2, \quad (10)$$

where the following notations have been introduced: $\hbar\omega_0 = 2 me^2$, $a = \pi^2 ch/k \approx 0.72$. We should also be concerned about the fulfillment of condition (1), i.e. $\omega \leq CN_e^{1/3}$. Thus, as an example of the fact that equation (10) can be fulfilled when $N_e^{1/3} a/T = 15$, we then obtain $N_e < 1.7 \cdot 10^{33} (\omega_0/\omega)^6$. This inequality is satisfied if we take $\omega \sim \omega_0$ and $N_e \sim N_0$. But if $N_e^{1/3} a/T = 10$, then from (10) we obtain $N_e < 4.4 \cdot 10^{39} (\omega_0/\omega)^6$. From the examples cited we can see that under the condition $a/T N_e^{1/3} \leq 15$ the process $e^+ + e^- \rightarrow \gamma$

* See Report I, DANArmSSR (Reports of the AS Arm SSR), Vol. XXX, No. 1 (1960).

with participation of the medium is entirely possible, while at lower temperatures, when $T < a/T N_e^{-1/3}$, this process is impossible. Under the same physical conditions, when the process $e^+ + e^- \rightarrow \chi$ is possible, the reverse process $\chi \rightarrow e^+ + e^-$ is forbidden by Pauli's principle; however it takes place as soon as free spaces appear in the propagation of electrons on account of various possible processes.

The dispersion will be determined by neutrons when

$$3.6 \cdot 10^{17} \left(\frac{N_e}{N_e} \right)^{2/3} \exp \left(-\frac{2a}{3T} N_e^{1/3} \right) \leq 3.6 \cdot 10^{17} \left(\frac{\omega_0}{\omega} \right)^2 < N_e^{1/3} \exp \left(-\frac{a}{T} N_e^{1/3} \right) \quad (11)$$

Here on the left-hand side the condition of applicability of taking the medium $\omega \ll aN_e^{1/3}$ is written. The right-hand side is the condition (9), with (30)^{*} also taken into consideration. As an example let us again take $a/T N_e^{1/3} = 15$ (the condition of degeneration of the electronic gas); then we obtain,

$$1.2 \cdot 10^{20} \left(\frac{N_e}{N_e} \right)^2 \leq 1.3 \cdot 10^{20} \left(\frac{\omega_0}{\omega} \right)^6 < N_e. \quad (11')$$

Here the left-hand side of the inequality is automatically fulfilled when the right-hand side is fulfilled. (11') is fulfilled when $N_e \geq 1.4 \cdot 10^{26}$ and $\omega \geq 7\omega_0$; the temperature of the corresponding regions of the star will here be $T \approx 10^8$ degrees. Hence when $a/T N_e^{1/3} =$

* See Report I, DANArSSR (Reports of the AS Arz SSF), Vol. XXX, No. 1 (1960).

15 and $\omega \gtrsim 7\omega_0$, the dispersion of waves is determined by neutrons, while at frequencies lower than $7\omega_0$ it is determined by electrons.

In the case when the nucleonic gas is also degenerate, the density of the electrons depends comparatively weakly on the density of the neutrons. When $N_n < 10^{36}$, we have $N_e \sim 1/8N_0$, $cN_e^{1/3} \approx 0.5\omega_0$, so that the processes $e^+ + e^-$ with participation of the medium either do not proceed at all or can proceed at the very threshold of the phenomenon at $h\omega \approx 2 mc^2$. At densities $N_n \gtrsim 8 \cdot 10^{35}$, the densities of the electrons will be sufficient for generation and annihilation of the pairs. But before the presence or absence of this phenomenon can be confirmed it is necessary to explain as well at what frequencies the dispersion is determined by electrons and at what frequencies it is determined by neutrons. To solve this problem let us turn again to condition (9). According to (32)* and (9), the dispersion is determined by neutrons when

$$N_n > 2.4 \cdot 10^{38} \frac{1}{\gamma^2} \left[\frac{\alpha}{\pi} + \chi \left(\frac{N_n}{N_0} \right)^{2/3} \right]^2, \quad (12)$$

where $\gamma = \omega/\omega_0$. Condition (12), as always, should be accompanied by the condition of applicability of selection of the medium, i.e. $\omega \ll cN_n^{1/3}$.

* See Report I, DANArMSSR (Reports of the AS Arm SSR), Vol. XXX, No. 1 (1960).

The dispersion of waves is determined by electrons if

$$N_n < 2.4 \cdot 10^{38} \frac{1}{v^2} \left[\frac{\alpha}{\pi} + \chi \left(\frac{N_n}{N_0} \right)^{2/3} \right]^2. \quad (13)$$

Let us recall that here the density of the electrons is determined by formula (32').* As we see, when $N_n \leq 10^{36}$, the process $\gamma \Rightarrow e^+ e^-$ can proceed only at the threshold energy, while when $N_n \geq 10^{36} \text{ cm}^{-3}$ the necessary and sufficient (of course, consideration of the condition $\omega \ll cN_e^{1/3}$ is necessary) conditions for performance of this process already exist.

Hard Cerenkov radiation. On the basis of the material cited in the preceding section, we come to the conclusion that at a given density of the substance a certain limiting frequency exists, so that when frequencies are higher than it, dispersion of waves is determined no longer by electrons, as was the case until then, but by neutrons. According to (7) at such frequencies the refractive index is greater than one and practically has a constant value, independent of the frequency. Inasmuch as the refractive index of the medium is greater than one, it is clear that in such a medium charged particles moving with a velocity exceeding the phase velocity of light will emit Cerenkov radiation. According to the conditions (1) and (9) only

* See Report I, DANArSSR (Reports of the AS Arm SSR), Vol. XXX No. 1 (1960).

quanta satisfying the inequality

$$10^3 N_e^{1/3} N_n^{-1/3} < h\omega \leq hc N_n^{1/3}. \quad (14)$$

can be emitted. In this energy interval, the distribution of the number of quanta according to their energies is uniform. According to (32)* the Cerenkov effect appears at energies of the particles

$$E > 2.2 \cdot 10^{20} N_n^{-1/3} M c^2,$$

where $M c^2$ is the intrinsic energy of the particle.

In the case when the nucleonic gas is nondegenerate, the relationship between the densities N_e and N_n is given by formula (30).^{*} As an illustration let us take $\alpha/T N_e^{1/3} = 10$; then from (14) when $N_e = 10^{32}$ (i.e., $N_n = 2.2 \cdot 10^{36}$) we find that $20 < h\omega \leq 28$ mev, while when $N_e = 4.5 \cdot 10^{33}$ (i.e. $N_n = 10^{38}$) we have $10 < h\omega \leq 100$ mev.

In the case when the nucleonic gas is also degenerate, the relationship between N_e and N_n is determined by formula (32').^{*} For this case (14) takes the following form,

$$2.6 \cdot 10^{13} N_n^{1/3} \left[\frac{\alpha}{\pi} + \gamma \left(\frac{N_n}{N_e} \right)^{2/3} \right] < h\omega \leq hc N_n^{1/3}. \quad (14')$$

From this we find that the density of the neutrons should be $N_n \approx 8 \cdot 10^3$. When $N_n = 10^{36} \text{ cm}^{-3}$, we have from (14') $13 < h\omega \leq 20$ mev,

* See Report I, DANArmSSR (Reports of the AS Arm SSR), Vol. XXX, No. 1 (1960).

while when $N_n = 10^{38}$ we obtain $1.6 < \omega/\omega_0 \leq 100$ mev.

The results obtained above can be summarized thus.

1. In this paper the dispersive properties of a medium at extremely high densities and temperatures, and namely approximately under such physical conditions as can exist in white dwarfs, have been investigated.

If the electronic gas is not degenerate (which seems rather improbable to us), then the refractive index of the medium depends on the temperature and is expressed by formula (26).* At electron densities $N_e \gg 10^{32} \text{ cm}^{-3}$, processes of single photon annihilation and generation of electron pairs will take place in the medium.

2. If under the physical conditions considered the temperatures are so low that the electronic gas is degenerate, then when the condition $\omega^2 N_n < 10^{60} N_e^{2/3}$ is observed (N_n is the density of the neutrons), the dispersion of the electromagnetic waves will be determined by the electrons. The index of refraction in this case is determined by formula (22).*

3. If the electronic gas is degenerate, while the nucleonic gas is not, then the relationship between the densities of the neutrons and electrons is determined by formula (30).* In this case under definite, completely fulfilled conditions, namely when condition (10) is

* See Report I, DANArmSSR (Reports of the AS Arm SSR), Vol. XXIX, No. 1 (1950).

observed, the processes $\gamma \rightleftharpoons e^+ + e^-$ can proceed with participation of the medium.

When the nucleonic gas is also degenerate, the relationship between the densities of the electrons and neutrons is determined by formula (32'). * When $N_n \leq 10^{36} \text{ cm}^{-3}$, the density of the electrons does not depend on the density of the neutrons and has a constant value, equal approximately to $1.5 \cdot 10^{31}$. Here, obviously, the processes $\gamma \rightleftharpoons e^+ + e^-$ with participation of the medium are impossible, or if possible, then only at the threshold energy $\hbar\omega = 2 mc^2$. At densities of the neutrons $N_n \geq 10^{36} \text{ cm}^{-3}$, the density of the electrons begins to increase with an increase in N_n , and this process will take place with participation of the medium. The conditions for the existence of this phenomenon are determined by formula (13).

4. When the electronic gas is degenerate, then at comparatively high densities and wave frequencies $\omega^2 n_n \geq 10^{60} N_e^{2/3}$, the dispersing properties of the medium are determined not by the electrons but by the neutrons. The essential wave frequencies and neutron densities at which this takes place, in the case of a nondegenerate nucleonic gas, are determined by condition (11), while in the case of degeneration -- by condition (12).

5. In the case when the dispersion of waves is determined by

* See Report I, DANArmSSR (Reports of the AS Arm SSR), Vol. XXX, No. 1 (1960).

neutrons, the index of refraction of the medium is greater than one and is determined by formula (7). But this means that, if the charged particle moves in such a medium with a velocity exceeding the phase velocity of light, then it will emit and absorb γ Cerenkov radiation. The spectrum of Cerenkov radiation is determined by formula (14). In the case of a degenerate nucleonic gas, the energies of the quanta are determined by condition (14').

In conclusion I shall express my thanks to academician V. A. Ambartsumyan for his valuable comments, and also to N. M. Kocharyan, I. I. Gol'dman, G. M. Garibyan, and A. Ts Amatun' for their numerous discussions and their manifestation of interest in the work.

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